

1 Recurrence Relations

1.1 Concepts

1. In order to solve a linear homogeneous we can replace the equation with its characteristic polynomial. For instance, the characteristic polynomial of $a_n = 2a_{n-1} + a_{n-2}$ is $\lambda^2 = 2\lambda + 1$. Then if $\lambda_1, \dots, \lambda_k$ are roots of this polynomial, then the general form of the solution is $a_n = C_1\lambda_1^n + \dots + C_k\lambda_k^n$.

1.2 Problems

2. True False If a_n, b_n are two solutions to a linear homogeneous equation, then $a_n + b_n$ is also a solution.
3. True False If a_n is a solution to a linear homogeneous equation, then ca_n is also a solution for any constant c .
4. Solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$ with $a_1 = 5, a_2 = 13$.
5. Find a recurrence relation such that $a_n = 2^n - 3^n$ is a solution to it. (Hint: What would the characteristic polynomial be?)
6. Solve the recurrence relation $a_n = 3a_{n-1} + 4a_{n-2}$ with $a_0 = 3$ and $a_1 = 2$.
7. Find a recurrence relation such that $a_n = n(-1)^n$ is a solution to it.
8. Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2}$ with $a_0 = 3$ and $a_1 = 4$.